AS-2796

Bsc. (Fifth Semester) Examination, 2013

Electronics

Paper: Second

(Wave Propagation)

Time Allowed: Three hours

Maximum Marks: 30

Note: Question no. 1 is compulsory. Attempt any five question from Section-B.

Section-A

Note: Attempt all questions. Each question carries 1 marks

1. Choose the correct answer:

(i) Velocity of propagation of an EM wave is

(a) $\sqrt{\mu_0 / \varepsilon_0}$ (b) μ_0 / ε_0 (c) $1/\sqrt{\mu_0 \varepsilon_0}$ (d) ε_0 / μ_0

Sol. (c)

(ii) For free space

(a) $\sigma = \infty$ (b) $\sigma = 0$ (c) $J \neq 0$ (d) $\mu_r = 0$

Sol. (b)

(iii) Dispersive power of gases varies inversely as the

(a) cube of wavelength (b) square of wavelength (c) wavelength (d) None

Sol. (a)

(iv) Index of refraction of gases under normal conditions is approximately

(a) infinity (b) 0 (c) unity (d) None

Sol. (c)

(v) Electromagnetic waves is

(a) Transverse in nature (b) longitudinal in nature (c) dual in nature (d) None

Sol. (a)

(vi) Refractive index of conducting medium will be 'n' is

(a) v/c (b) c/v (c) c.v (d) None

Sol. (b)

(vii) Transmission lines is made up of

(a) Perpendicular cylindrical conductor (b) parallel cylindrical conductors (c) perpendicular cylindrical semiconductor (d) parallel cylindrical insulator

Sol. (b)

(viii) The general transmission line equation correlates the voltages and current at the ___ ends of the line

(a) Four (b) one (c) three (d) two

Sol. (d)

(ix) Reflection coefficient in transmission line is shown by

(a) Γ (b) β (c) α (d) θ

Sol. (a)

(x) Input impedance of the transmission line is Z_{in} is

(a) V_{in}/I_{in} (b) I_{in}/V_{in} (c) Vs/Is (d) Is/Vs

Sol. (c)

Section-B

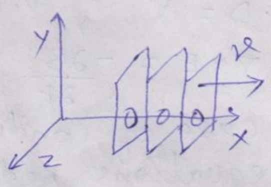
Q.2. Derive expression for monochromatic plane waves in vacuum and show that E and B are in phase and mutually perpendicular.

Soln. The electric and magnetic field vectors E and B in empty space, satisfy the 3-D wave equation

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}, \quad \nabla^2 B = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} \quad —①$$

where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ is the speed of light in vacuum.

We consider sinusoidal waves of frequency ω. Such waves are called monochromatic, Suppose the waves are travelling in the x-direction and have no y- or z-dependence. These are called plane waves, because the fields are uniform over every plane perpendicular to the direction of propagation in figure.



$$\tilde{E}(x,t) = \tilde{E}_0 e^{i(kx-\omega t)}$$
$$\tilde{B}(x,t) = \tilde{B}_0 e^{i(kx-\omega t)} \quad —②$$

k is wave number.

$\tilde{E}_0$ and $\tilde{B}_0$ are the complex amplitudes of the electric and magnetic fields. The physical fields are the real parts of $\tilde{E}$ and $\tilde{B}$.

Since $\nabla \cdot E = 0$ and $\nabla \cdot B = 0$, it follows that $(\tilde{E}_0)_x = (\tilde{B}_0)_x = 0$.

This implies that electromagnetic waves are transverse.

The electric and magnetic fields are perpendicular to the direction of propagation.

Faraday's law, $\nabla \times E = -\frac{\partial B}{\partial t}$ implies a relation between the electric and magnetic amplitudes.

$$-K(\tilde{E}_0)_z = \omega(\tilde{B}_0)_y, \quad K(\tilde{E}_0)_y = \omega(\tilde{B}_0)_z \quad (4)$$

$$\tilde{B}_0 = \frac{K}{\omega}(\hat{z} \times \tilde{E}_0) \quad (5)$$

E and B are in phase and mutually perpendicular. The real amplitudes of E and B are related by

$$B_0 = \frac{K}{\omega} E_0 = \frac{1}{c} E_0 \quad (6)$$

Q.3. In propagation of electromagnetic waves in Conducting medium show that the velocity of wave, being inversely proportional to σ is very small in good conductor.

Soln. We know that maxwell's equations are

$$\nabla \cdot D = \rho, \quad \nabla \cdot B = 0, \quad \nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad [J = \sigma E, D = \epsilon E, B = \mu H]$$

In case of conducting media as $\rho = 0$, so the field equations reduces to

$$\left.\begin{aligned} \operatorname{div} E &= 0 \quad (a) \\ \operatorname{div} H &= 0 \quad (b) \\ \operatorname{curl} H &= \sigma E + \epsilon \frac{\partial E}{\partial t} \quad (c) \\ \operatorname{curl} E &= -\mu \frac{\partial H}{\partial t} \quad (d) \end{aligned}\right\} \quad (1)$$

Now if we take curl of equation (c), then

$$\nabla \times (\nabla \times H) = \nabla \times \left[\sigma E + \epsilon \frac{\partial E}{\partial t}\right]$$

$$\text{grad} \cdot \operatorname{div} H - \nabla^2 H = \sigma(\nabla \times E) + \epsilon \frac{\partial}{\partial t}(\nabla \times E) \quad (2)$$

But from equation (b) and (d)

$$\nabla \cdot H = 0 \quad \text{and} \quad \nabla \times E = -\mu \frac{\partial H}{\partial t}$$

So equation (2) reduces to

$$-\nabla^2 H = -\sigma \mu \frac{\partial H}{\partial t} - \epsilon \mu \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H - \sigma\mu \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{— (A)}$$

Now taking curl of eq. (d), then

$$\nabla \times (\nabla \times E) = -\mu \nabla \times \frac{\partial H}{\partial t}$$

$$\text{grad div} E - \nabla^2 E = -\mu \frac{\partial}{\partial t}(\nabla \times H) \quad \text{— (3)}$$

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \sigma\mu \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{— (B)}$$

Equation (A) and (B) are known as equation of telegraphy.

$$\nabla^2 \psi - \sigma\mu \frac{\partial \psi}{\partial t} - \mu\epsilon \frac{\partial^2 \psi}{\partial t^2} = 0 \quad \text{— (4)}$$

$$\psi = \psi_0 e^{-i(\omega t - k \cdot r)}$$

$$\begin{Bmatrix} E \\ H \end{Bmatrix} = \begin{Bmatrix} E_0 \\ H_0 \end{Bmatrix} e^{-i(\omega t - k \cdot r)}$$

Taking $ik^* \to \nabla$ and $-i\omega \to \frac{\partial}{\partial t}$

And so equations (B) and (A) yield

$$(-k^{2*} + i\omega\sigma\mu + \mu\epsilon\omega^2)\begin{Bmatrix} E \\ H \end{Bmatrix} = 0$$

$$k^{2*} - i\sigma\mu\omega - \mu\epsilon\omega^2 = 0$$

$$k^{2*} = \mu\epsilon\omega^2 \left[1 + \frac{i\sigma}{\epsilon\omega}\right] \quad \text{— (5)}$$

Equation (5) shows that propagation constant is complex and may be expressed as

$$k^* = \alpha + i\beta \quad \text{— (6)}$$

$$k^{2*} = \alpha^2 - \beta^2 - i2\alpha\beta \quad \text{— (7)}$$

Comparing equation (5) and (7) we get

$$\alpha^2 - \beta^2 = \mu\epsilon\omega^2 \quad \text{— (8)}$$

$$2\alpha\beta = \sigma\mu\omega \quad \text{— (9)}$$

Putting value of β from (9) in (8)

$$\alpha^2 - \frac{\mu^2\sigma^2\omega^2}{4\alpha^2} = \mu\epsilon\omega^2 \Rightarrow \alpha^4 - \mu\epsilon\omega^2\alpha^2 - \frac{\mu^2\sigma^2\omega^2}{4} = 0$$

$$\alpha^2 = \frac{\mu\epsilon\omega^2 \pm \sqrt{[(\mu\epsilon\omega^2)^2 + \mu^2\sigma^2\omega^2]}}{2}$$

$$\alpha = \pm\,\omega\sqrt{\frac{\mu\epsilon}{2}}\left[1 \pm \sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}\right]^{1/2} \quad \text{—— (10)}$$

Now as in the limit $\sigma \to 0$, $\mu \to \mu_0$, $\epsilon \to \epsilon_0$ and so equation (5) and (6) reduces as

$$k^{2*} = \mu_0\epsilon_0\omega^2 \text{ and } k^* = \alpha$$

so that $\alpha \to \omega\sqrt{\mu_0\epsilon_0}$

This is turn implies that correct value of α given by equation (10) is

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[1+\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}\right]^{1/2}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{\left\{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2\right\}}+1\right]^{1/2} \quad \text{—— (D)}$$

Similarly putting the value of α in turns of β from equation (9) in (8) and solving we get

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{\left\{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2\right\}}-1\right]^{1/2} \quad \text{—— (E)}$$

and in terms of these values of α, β the ~~vector~~ field vectors given by equation (C) may be expressed as

$$\left\{\frac{E}{H}\right\} = \left\{\frac{E_0}{H_0}\right\} e^{i[\omega t - (\alpha + i\beta)n\cdot r]}$$

$$\left[E/H\right\} = \left\{\frac{E_0}{H_0}\right\} e^{-\beta n\cdot r} \cdot e^{-i(\omega t - \alpha n\cdot r)} \quad \text{—— (11)}$$

From equation (11) it is clear that —

(i) The phase velocity of electromagnetic wave is

$$v = \frac{\omega}{\alpha} = \sqrt{\frac{2}{\mu\epsilon}}\left[\sqrt{\left\{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2\right\}}+1\right]^{-1/2} \quad \text{—— (12a)}$$

which for $\sigma/\epsilon\omega >> 1$ reduces to

$$v = \sqrt{(2/\mu\epsilon)}\left(\frac{\sigma}{\epsilon\omega}\right)^{-1/2} = \sqrt{\frac{2\omega}{\mu\sigma}} \quad \text{—— (12b)}$$

i.e. the velocity of wave, being inversely proportional to σ is very small in good conductor.

Q.4. What is dispersion? Explain normal and anomalous dispersion.

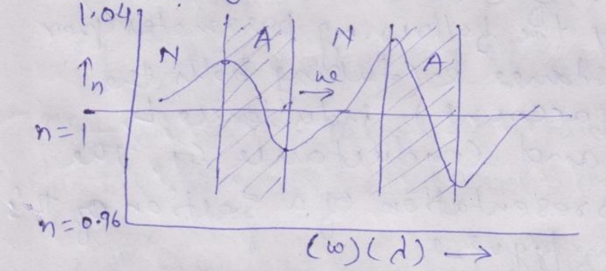
Sol^n. If in medium the index of refraction varies with frequency (i.e. wavelength) then the medium is said to be dispersive. The phenomenon itself is called dispersion and rate of change of refractive index with wavelength i.e. $dn/d\lambda$ is known as dispersive power.

Generally the variation of 'n' is such that —

(i) The index of refraction increases as the frequency increases.

(ii) The rate of increase $dn/d\omega$ i.e. the slope of the n-ω curve is greater at high frequencies.

However it is also found that ones small frequency range there is often decrease of index of refraction with the increase in frequency.



A → Regions in which the dispersion is anomalous.

N → Regions in which the dispersion is normal.

In these narrow spectral region due to its anomalous. Normal and Anomalous dispersions are shown in figure.

Normal Dispersion: —

If $\gamma_{0j} \to 0$ and $\omega < \omega_{0j}$ i.e. the region is remote from the natural frequencies of the electrons equation (A) i.e. $n^2 = 1 + \frac{1}{4\pi\epsilon_0} \cdot \frac{4\pi Ne^2}{m} \cdot \Sigma \frac{f_j}{(\omega_{0j}^2 - \omega^2) - i\gamma_{0j}\omega}$ —— (A) reduces to

$$n^2 = \left[1 + \frac{1}{4\pi\epsilon_0}\frac{4\pi Ne^2}{m}\Sigma\frac{f_j}{\omega_{0j}^2 - \omega^2}\right] \text{ —— (b)}$$

From equation (b) it is clear that the refractive index is real and increases with frequency of the incident waves i.e. for a given medium red light has the lowest while violet largest index of refraction in the optical range of frequencies.

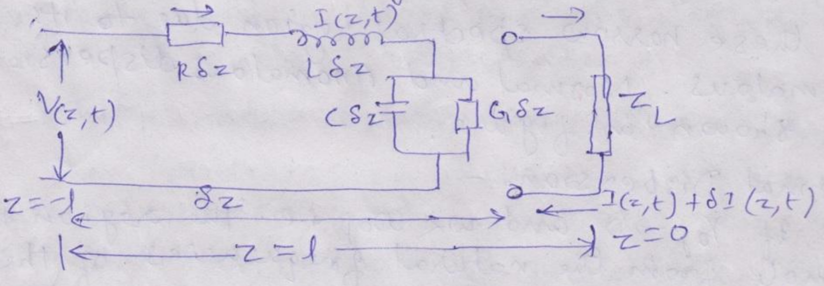
Anomalous Dispersion:-

If $\omega \simeq \omega_{0j}$ i.e. in the extremely narrow spectral region in which the impressed frequencies include one of the so many natural frequencies of the electron. For simplicity we assume that there is one natural frequency i.e. $\omega_{0j} = \omega_0$ so that equation (A) becomes

$$n^{*2} = 1 + \frac{1}{4\pi\epsilon_0} \frac{4\pi N e^2}{m} \cdot \frac{1}{(\omega_0^2 - \omega^2) - i\gamma_0 \omega}$$

Q.5. Derive transmission line equations correlating the voltages and the currents at the two ends of the line.

Sol^n. We consider the transmission line cable as a pair of conductors of circular cross-section which is characterized by the following parameter per unit length; resistance R (taking both the conductors into account), inductance L, capacitance C, and conductance G. The ~~most~~ circuit representation of a section of this line is shown in figure.



Considering the voltage and the current relations along the line, as shown in figure i.e.

$$\left(\frac{\partial I}{\partial z}\right) + C\left(\frac{\partial V}{\partial t}\right) = 0 \text{ and } \frac{\partial V}{\partial z} + L\left(\frac{\partial I}{\partial t}\right) = 0$$

$V(z,t) - R\,\delta z\, I(z,t) - L\,\delta z\left(\frac{\partial}{\partial t}\right)(I(z,t)) = V(z,t) + \delta V(z,t)$

and $I(z,t) - G\,\delta z\,[V(z,t) + \delta V(z,t)] - C\,\delta z\left(\frac{\partial}{\partial t}\right)[V(z,t) + \delta V(z,t)] = I(z,t) + \delta I(z,t)$

Simplifying and ignoring $(\delta z)^2$ and higher degree terms,

$$\left(\frac{\partial}{\partial z}\right)V(z,t) + R I(z,t) + L\left(\frac{\partial}{\partial t}\right)I(z,t) = 0 \quad \text{— (1)}$$

$$\left(\frac{\partial}{\partial z}\right)I(z,t) + G V(z,t) + C\left(\frac{\partial}{\partial t}\right)V(z,t) = 0 \quad \text{— (2)}$$

The most important case for practical problems is when $V(z,t)$ and $I(z,t)$ vary sinusoidally with time. So we can write:

$$I(z,t) = \mathrm{Re}[I(z)\exp(j\omega t)] \text{ and}$$

$$V(z,t) = \mathrm{Re}[V(z)\exp(j\omega t)] \quad \text{— (3)}$$

where $I(z)$ and $V(z)$ are complex amplitudes of the current and the voltage along the line.

Hence equations (2) become

$$\left(\frac{d}{dz}\right)V(z) + (R + j\omega L)I(z) = 0 \quad \text{— (4)}$$

$$\left(\frac{d}{dz}\right)I(z) + (G + j\omega C)V(z) = 0 \quad \text{— (5)}$$

Differentiating the first equation w.r.t. z and combining with the second; and doing the vice versa, we get

$$\left(\frac{d^2}{dz^2}\right)V(z) + (R + j\omega L)(G + j\omega C)V(z) = 0 \quad \text{— (6)}$$

$$\left(\frac{d^2}{dz^2}\right)I(z) + (R + j\omega L)(G + j\omega C)I(z) = 0 \quad \text{— (7)}$$

both of which are complex one dimensional wave equations.

Let $(R + j\omega L)(G + j\omega C) = \gamma^2 = (\alpha + j\beta)^2$ — (8)

which gives $\alpha = \frac{1}{\sqrt{2}}\left[\sqrt{(R+j\omega L)(G+j\omega C)} + (RG - \omega^2 LC)\right]^{1/2}$ — (9)

$$\beta = \frac{1}{\sqrt{2}}\left[\sqrt{(R+j\omega L)(G+j\omega C)} - (RG - \omega^2 LC)\right]^{1/2} \quad \text{— (10)}$$

where γ is the complex propagation constant, α is ~~attenut~~ attenuation constant and β is the phase constant.

Equation (6) and (7) then become

$$\left(\frac{d^2}{dz^2}\right) V(z) + \gamma^2 V(z) = 0 \quad \text{--- (11)}$$

$$\left(\frac{d^2}{dz^2}\right) I(z) + \gamma^2 I(z) = 0 \quad \text{--- (12)}$$

The solution of both these equations is the same i.e.

$$V(z) = V_1 \exp(-\gamma z) + V_2 \exp(\gamma z) \quad \text{--- (13)}$$

$$I(z) = I_1 \exp(-\gamma z) + I_2 \exp(\gamma z) \quad \text{--- (14)}$$

where V_1, V_2, I_1 and I_2 are unknown constants.

Expressing the solutions of equation (13) and (14) in hyperbolic form, we have

$$V = A_1 \cosh(\gamma z) + B_1 \sinh(\gamma z) \quad \text{--- (15)}$$

$$I = A_2 \cosh(\gamma z) + B_2 \sinh(\gamma z) \quad \text{--- (16)}$$

where the unknowns A_1, B_1, A_2, B_2 are determined by the use of boundary conditions at the receiving end ($z=0$) and at the sending end ($z=-l$), as shown: (a) at $z=0$, $V=V_R$, $I=I_R$ and

(b) at $z=-l$, $V=V_S$, $I=I_S$ --- (17)

Substituting these equations in (15) + (16) and also using equation (4) and (5), we get

$$V_S = V_R \cosh(\gamma l) + Z_C I_R \sinh(\gamma l) \quad \text{--- (18)}$$

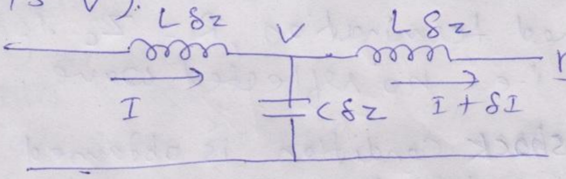
$$I_S = I_R \cosh(\gamma l) + \left(\frac{V_R}{Z_C}\right) \sinh(\gamma l) \quad \text{--- (19)}$$

These are general transmission line equations correlating the voltages and the currents at the two ends of the line.

Q.6. Write notes on transmission line.

soln. The transmission lines, most often met with in practice, are made up of two (very) nearly parallel, cylindrical conductors. Since the conductor shapes can never be perfect, an exact rigorous analysis would be rather difficult. Hence, instead of treating the lines as a boundary value problem, The theory of travelling waves in lines (and also in cables) can be developed from the circuit standpoint. The lines are represented as a complicated combination of resistors, capacitors, and inductors. So, instead of the electric ~~to~~ and magnetic field intensities at all points where the fields of the lines exist, we consider the unknown voltage between the two lines and the current through them.

We consider the cable as a pair of conductors, in which each length δz has the inductance $L\delta z$, and between the conductors in each length δz is the capacitance $C\delta z$. So, at a point A on the line (where its voltage is V).



The charge in the capacitor is $= CV\delta z$ and the current in the capacitor = the rate of increase of the charge $= C\left(\frac{\partial V}{\partial t}\right)\delta z$

Equating the inflowing and the outflowing currents at the point A (referring to figure)

$$I = C\left(\frac{\partial V}{\partial t}\right)\delta z + (I + \delta I)$$

$$\delta I + C\left(\frac{\partial V}{\partial t}\right)\delta z = 0$$

$$\left(\frac{\partial I}{\partial z}\right) + C\left(\frac{\partial V}{\partial t}\right) = 0 \quad (1)$$

Now, considering the potentials along the line, potential at the point A = potential at the point $B = L\delta z\left(\frac{\partial}{\partial t}\right)(I+\delta I) = -\delta V$

$L\delta z\left(\frac{\partial I}{\partial t}\right) = -\delta V$, neglecting higher order terms. In the limit, $\left(\frac{\partial V}{\partial z}\right) + L\left(\frac{\partial I}{\partial t}\right) = 0$ — (2)

Eliminating I between equations (1) and (2), we get $\left(\frac{\partial^2 V}{\partial z^2}\right) = \left(\frac{1}{u^2}\right)\left(\frac{\partial^2 V}{\partial t^2}\right)$, where $u^2 = \frac{1}{LC}$ — (3)

which is the usual one-dimensional wave equation.

Q.7. Discuss the three cases open circuit, short circuit and matched termination in transmission line's with negligible losses.

Solⁿ. there are three limiting cases of this stage, i.e. (a) open circuit (O.C.) termination

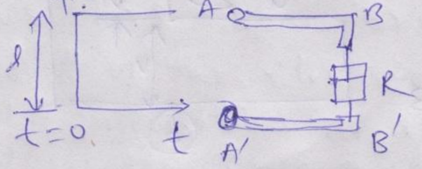
$R \to \infty$ $\therefore$ $V' = V$ and $I' = I$

(b) Short circuit (S.C.) termination $R = 0$

$\therefore V' = -V$ and $I' = I$

(c) Matched termination, $R = Z_c$ $\therefore$ $V' = 0$ and $I' = 0$, i.e. no reflected wave.

Transient shock condition is obtained when a switch in the line is closed suddenly. ~~Sud~~ Sudden switching and lightening are the two main sources, which cause the transient behaviours of the circuits. This is equivalent to the ~~appl~~ application of a unit step function to the line. This function is a wave which suddenly rises from $V=0$ to $V=1$ at the time $t=0$, and the remains constant at that value thereafter figure. Any other types of shocks can be built up by a successive superimposition of the unit steps. Let a voltage step of the type mentioned be applied to the terminals AA' of a cable, whose other end BB' is terminated by a resistance R as shown in figure.



(i) The first effect of this step function is that a voltage wave of magnitude V and a current (V/Zc) travel along the cable from A and B with velocity u.

On reaching B, a reflected wave of voltage V' and a current (V'/Zc) starts travelling from B to A. The magnitude of V' is determined by R such that (voltage/current) at BB' = R.

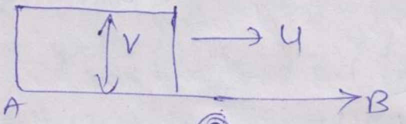
At this stage,
Potential drop across BB' = V+V' and the current flowing out at B = $\frac{V}{Z_c} - \frac{V'}{Z_c}$

$$\frac{V+V'}{\frac{V-V'}{Z_c}} = R \Rightarrow \text{ and hence } \quad V' = \left(\frac{R-Z_c}{R+Z_c}\right)V \quad \text{— (8)}$$

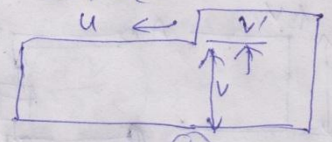
The reflected wave on reaching AA', meets the generator impedance, which produces the unit function p.d.; this is ideally a zero impedance, but infact negligibly small impedance. Hence this returning wave meets a short circuit (S.C.) condition at AA'. Thus, for the next stage, a wave −V' travels from A to B, as shown in figure.

On reaching B, it is reflected back according to figure and the reflected wave is

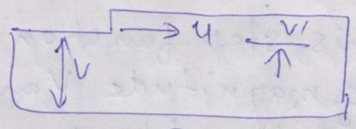
$$= -\left(\frac{R - Z_C}{R + Z_C}\right)V' = -\left(\frac{R - Z_C}{R + Z_C}\right)^2 V$$ shown in figure (d).



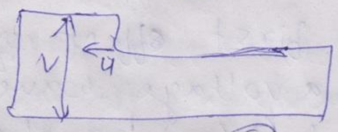
(a)



(b)



(c)



(d)

Q.8. A coaxial cable has Z_0 of $75\,\Omega$ and a capacitance of 70 pF/m. Find its inductance per meter.

Soln. Given, $Z_0 = 75\,\Omega$, capacitance $C = 70\,pF/m$

$C = 70 \times 10^{-12}\,F/m$

we know that

Impedance $Z_0 = \sqrt{L/C}$

$\frac{L}{C} = Z_0^2$, $Z_0^2 C = L$

$L = (75)^2 \times 70 \times 10^{-12} = 5625 \times 70 \times 10^{-12}$

$= 393750 \times 10^{-12}$

$= 0.3937\ \mu H/m$

$\therefore L = 0.3937\,\mu H/m$

Inductance (L) is $0.3937\ \mu H/m$.